

Higher-curvature Corrections to Holographic Entanglement with Momentum Relaxation

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Abstract

We study the effects of Gauss-Bonnet corrections on entanglement entropy and mutual information in the holographic model with momentum relaxation. There are in fact two kinds of deformation in the states of conformal field theory in this model: the higher-curvature terms, which could address the low-energy quantum excitation corrections, and the deformation due to scalar fields, which are responsible for the momentum conservation breaking. We use holographic methods to obtain the corresponding changes due to these deformations in the finite and universal parts of entanglement entropy for strip geometry. Holographic calculation indicates that mutual and tripartite information undergo a transition beyond which they identically change their values. We find that the behavior of transition curves depends on the sign of the Gauss-Bonnet coupling λ . The transition for $\lambda > 0$ takes place in larger separation of subsystems than that of $\lambda < 0$.

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1 Introduction

The Anti-de Sitter (AdS)/Conformal Field Theory (CFT) correspondence postulates a relationship between quantum physics of strongly correlated many-body systems and the classical dynamics of gravity which lives in one higher dimension [1]. Through this correspondence, a great deal of progress has been made in understanding the dynamics of strongly coupled gauge theories, and it has been also further extended to cover topics related to the condensed matter theory [2–4]. In this context, the solutions of Einstein-Maxwell-Dilaton theories have been frequently employed to address the states of underlying field theory. However, since such charged solutions are translational invariant, a small perturbation such as turning on an electric field, could result in an infinite DC conductivity. It is obvious that such a model cannot present a realistic description of aforementioned systems and it is argued that momentum must be relaxed in these kinds of theories to avoid such a strange behavior. This can be done, for example, by breaking the translational invariance property [5–12]. In this direction, Andrade and Withers presented a simple holographic model for momentum relaxation [13]. Their model consists of Einstein-Maxwell theory in $(d + 1)$ -dimensional bulk space together with $d - 1$ massless scalar fields. The neutral scalar fields in the bulk theory are dual to some specific operators with spatially dependent sources and, in principle, these spatial sources can be chosen in a way that the bulk stress tensor and hence, the resulting black brane geometry are homogeneous and isotropic. Momentum relaxation concept is realized through these spatially dependent sources. Scalar fields lead to deformed state at the CFT side and different aspects of this deformation have been considered in several papers, *e.g.*, some non-local measures of entanglement in such a model have been recently studied in [14] via the holographic methods.

In this paper, we use this model and investigate the effect of including higher-order curvature terms in the gravitational action on holographic entanglement entropy (HEE); in this direction, we also study the mutual and tripartite information. In the holographic models, considering higher-curvature terms in the gravity action is well motivated for reasons; in particular, addressing different types of central charges could be an example [15–20]. However, in general, higher-derivative terms

could potentially introduce ghost degrees of freedom, but it is known that a special combination of curvature squared terms, namely the Einstein Gauss-Bonnet (GB) theory leads to second order equations of motion and the theory is free of ghosts. Holographically, GB term plays the role of leading-order corrections to the Einstein gravity and in the context of AdS/CFT, GB background is dual to a theory with different central charges, *i.e.*, a and c functions [21]. In the model considered in this work, the gravitational action contains five-dimensional GB term together with three specific spatially dependent massless scalar fields, and we are going to study the change of HEE for strip entangling region due to these corrections.

To compute HEE in the Einstein's theory of gravity, there is an elegant proposal made by Ryu and Takayanagi (RT) [22]. According to the RT proposal, for a definite entangling region in the boundary, the entanglement entropy is related to the minimal surface¹ in the bulk whose boundary coincides with the boundary of the entangling region. However, the RT proposal works for quantum field theories dual to Einstein gravity and the corresponding CFTs contain only one independent central charge. To study general field theories in the context of holography, higher-derivative terms are in fact needed at the gravity side. Therefore, in order to compute HEE in the semi-classical regime when some higher-order derivative terms are added into the Einstein gravity, RT proposal should be replaced by some other recipes [16, 24–27]. Some related works in this subject can also be found, for example in [28–31] and references therein.

In this paper, we will follow the proposal of [25] to study the HEE which will be the subject of section 2. We will restrict our discussion to GB gravity theory with momentum relaxation and compute the HEE for strip entangling region in section 3. In section 4, some other measurements of quantum entanglement, *i.e.*, mutual and tripartite information and their quantum phase transitions will be considered. Precisely, we are interested in the effect of GB corrections to these phase transitions and the monogamy of mutual information in holographic theories with momentum relaxation. Finally, the subject is concluded in the last section.

2 Holographic Entanglement Entropy in Higher-order Theories

Entanglement entropy is an important non-local measure of different degrees of freedom in a quantum mechanical system [32]. This quantity similar to other non-local quantities, *e.g.*, Wilson loop and correlation functions, can also be used to classify the various quantum phase transitions and critical points of a given system [33].

To define entanglement entropy in its spatial (or geometric) description, let us divide a constant time slice into two spatial regions A and B where they are complement to each other. Thus, the corresponding total Hilbert space can be written in a specific partitioning as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. By integrating out the degrees of freedom that live in the complement of A , the reduced density matrix for region A can be computed as $\rho_A = \text{Tr}_B \rho$ where ρ is the total density matrix. The entanglement

¹In the extended version of RT proposal named as HRT proposal, for time-dependent geometries, one should use the extremal surface [23].

entropy is given by the Von-Neumann formula for this reduced density matrix as follows

$$S = -\text{Tr } \rho_A \log \rho_A. \quad (2.1)$$

For local d -dimensional quantum field theories, entanglement entropy follows the area law and it is infinite; the structure of the infinite terms are generally as follows [16, 34, 35]

$$S(V) = \frac{g_{d-2}(\mathcal{A}_A)}{\epsilon^{d-2}} + \cdots + \frac{g_1(\mathcal{A}_A)}{\epsilon} + g_0(\mathcal{A}_A) \ln \epsilon + s(V), \quad (2.2)$$

where ϵ is the UV cutoff, \mathcal{A}_A and V stand for the area and volume of the entangling region in the boundary, $s(V)$ is the finite part of entropy and $g_i(\mathcal{A}_A)$ are local and extensive functions on the boundary of entangling region, which are homogeneous of degree i . The coefficient of the most divergent term is proportional to the area of the entangling surface and this is indeed the area law which is due to the infinite correlations between degrees of freedom near the boundary of entangling surface. The coefficients of infinite terms are not physical whereas the coefficient of logarithmic term is physical and universal in a sense that it is not affected by cutoff redefinitions.

Although computing the entanglement entropy in the context of field theory is indeed a difficult task, thanks to the AdS/CFT correspondence one can use RT proposal to find HEE, as mentioned in the introduction; this proposal defines HEE in terms of the minimal area of codimension two hypersurface in the bulk. However, for actions with higher-derivative terms, one should use other proposals to compute HEE; for example, in the case of curvature squared terms with the following action

$$\mathcal{I} = \frac{1}{16\pi G_N} \int_M d^{d+1}x \sqrt{-g} \left[R - 2\Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{1}{2} \sum_{i=1}^{d-1} (\partial\phi_i)^2 \right], \quad (2.3)$$

pursuing the proposal of [25], HEE is given by

$$S = \frac{A(\Sigma)}{4G_N} + \frac{1}{4G_N} \int_{\Sigma} \sqrt{\sigma} d^{d-1}x \left[2aR + b \left(R_{\mu\nu} n_i^{\mu} n_i^{\nu} - \frac{1}{2} \sum_i \left(\text{Tr} \mathcal{K}^{(i)} \right)^2 \right) \right. \\ \left. + 2c \left(R_{\mu\nu\alpha\beta} n_i^{\mu} n_i^{\alpha} n_j^{\nu} n_j^{\beta} - \sum_i \mathcal{K}_{\mu\nu}^{(i)} \mathcal{K}_{(i)}^{\mu\nu} \right) \right]. \quad (2.4)$$

In the above equations, G_N stands for Newton's constant, the cosmological constant is $\Lambda = -\frac{d(d-1)}{2L_{AdS}}$, ϕ_i are the minimally coupled massless scalar fields, σ is the induced metric determinant, n_i ($i = 1, 2$) are the orthogonal normal vectors on the codimension two hypersurface Σ and $\mathcal{K}_{\mu\nu}^{(i)}$ are the extrinsic curvature tensors on Σ defined as

$$\mathcal{K}_{\mu\nu}^{(i)} = h_{\mu}^{\lambda} h_{\nu}^{\rho} (n_i)_{\lambda;\rho}, \quad h_{\mu}^{\lambda} = \delta_{\mu}^{\lambda} + \xi \sum_i (n_i)_{\mu} (n_i)^{\lambda}, \quad (2.5)$$

where ξ is $+1$ for time-like and -1 for space-like vectors. It is noted that the first term in (2.4) is just the RT formula.

Corresponding equations of motion of (2.3) are given by

$$\begin{aligned} \nabla_\alpha \nabla^\alpha \phi_i &= 0, \\ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(a R^2 + b R_{\alpha\beta} R^{\alpha\beta} + c R_{\alpha\beta\gamma\sigma} R^{\alpha\beta\gamma\sigma} \right) &+ 2a R_{\mu\nu} R - 4c R_\mu{}^\alpha R_{\nu\alpha} \\ &+ (2b + 4c) R^{\alpha\beta} R_{\mu\alpha\nu\beta} + 2c R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} + \left(2a + \frac{b}{2} \right) g_{\mu\nu} \nabla_\alpha \nabla^\alpha R + (b + 4c) \nabla_\alpha \nabla^\alpha R_{\mu\nu} \\ &- (2a + b + 2c) \nabla_\nu \nabla_\mu R + \sum_{i=1}^{d-1} \left(\frac{1}{4} g_{\mu\nu} \partial_\alpha \phi_i \partial^\alpha \phi_i - \frac{1}{2} \partial_\mu \phi_i \partial_\nu \phi_i \right) = 0. \end{aligned} \quad (2.6)$$

It is worth mentioning that the contribution of scalar fields to the stress tensor is supposed to be homogeneous, thus one gets a homogeneous and isotropic black brane solution. The solution can be written as

$$ds^2 = \frac{L_{AdS}^2}{\rho^2} \left(-f(\rho) dt^2 + \frac{1}{f(\rho)} d\rho^2 + dx_1^2 + dx_2^2 + dx_3^2 \right). \quad (2.7)$$

We will return to this solution later. In the following, we restrict our study to five-dimensional geometric background. In order to compute HEE, let us consider the following strip entangling region

$$-\frac{\ell}{2} < x_1 \equiv x < \frac{\ell}{2}, \quad -\frac{H}{2} < x_2 \text{ and } x_3 < \frac{H}{2}, \quad (2.8)$$

where we assume $H \gg \ell$ and H plays an infrared regulator distance along the entangling surface. The corresponding codimension two hypersurface in a constant time slice can be parametrized by $x_1 = x(\rho)$; therefore, the induced metric becomes

$$ds_{ind}^2 = \frac{L_{AdS}^2}{\rho^2} \left[\left(x'^2 + f^{-1} \right) d\rho^2 + dx_2^2 + dx_3^2 \right], \quad (2.9)$$

where the *prime* stands for the derivative with respect to ρ . Moreover, the two orthogonal normal vectors are obtained as follows

$$\begin{aligned} \Sigma_1 : t = 0 & \quad n_1 = \left\{ \frac{\sqrt{f} L_{AdS}}{\rho}, 0, 0, 0, 0 \right\}, \\ \Sigma_2 : x_1 - x(\rho) = 0 & \quad n_2 = \left\{ 0, -\frac{x' L_{AdS}}{\rho \sqrt{f x'^2 + 1}}, \frac{L_{AdS}}{\rho \sqrt{f x'^2 + 1}}, 0, 0 \right\}. \end{aligned} \quad (2.10)$$

The corresponding extrinsic curvatures of the hypersurface are given by

$$\mathcal{K}_{\mu\nu}^{(1)} = 0, \quad \mathcal{K}_{\mu\nu}^{(2)} = L_{AdS} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 f^{-1} & C_1 x' & 0 & 0 \\ 0 & C_1 x' & C_1 f x'^2 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & C_2 \end{pmatrix}, \quad (2.11)$$

where

$$C_1 = \frac{2(1+fx'^2)fx' - \rho(f'x' + 2fx'')}{2\rho^2(1+fx'^2)^{5/2}}, \quad C_2 = \frac{fx'}{\rho^2\sqrt{1+fx'^2}}. \quad (2.12)$$

Consequently, for a slab entangling region, the entanglement entropy of (2.4) becomes

$$S = \frac{H^2 L_{AdS}^3}{4G_N} \int d\rho \frac{\sqrt{x'^2 + f^{-1}}}{\rho^3} (1 + \mathcal{A} + \mathcal{B}), \quad (2.13)$$

where

$$\begin{aligned} \mathcal{A} &= \frac{-8(10a+2b+c)f + (32a+7b+4c)\rho f' - (4a+b)\rho^2 f''}{2L_{AdS}^2} + \frac{[(3b+4c)\rho f' - (b+4c)\rho^2 f'']fx'^2}{2L_{AdS}^2(1+fx'^2)}, \\ \mathcal{B} &= -\frac{\rho^4 [b(2C_2 + C_1(1+fx'^2))^2 + 4c(2C_2^2 + C_1^2(1+fx'^2)^2)]}{2L_{AdS}^2}. \end{aligned} \quad (2.14)$$

Now, the main task is to minimize the entropy functional of (2.13). In the case of the GB gravity we will come back to these equations and within this context, in the following section, we will study the entanglement entropy under the mentioned deformations of the bulk theory.

3 Five-dimensional GB Gravity with Momentum Relaxation

The GB gravity can indeed be obtained by setting $a = c = -\frac{b}{4} \equiv \frac{\lambda}{2}L_{AdS}^2$ in (2.3), where λ is a dimensionless coupling constant that controls the strength of the GB term. Thus, the five-dimensional Einstein-GB-scalar gravitational action with momentum relaxation is given by the following expression

$$\mathcal{I} = \frac{1}{16\pi G_N} \int_M d^5x \sqrt{-g} \left[R + \frac{12}{L_{AdS}^2} + \frac{\lambda L_{AdS}^2}{2} (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2) - \frac{1}{2} \sum_{i=1}^3 (\partial\phi_i)^2 \right]. \quad (3.1)$$

The GB gravity in five dimensions is itself important because in a given background, its equations of motion for a propagating perturbation contain only two derivatives.

The scalar fields are considered to be linearly dependent on spatial coordinates, *i.e.*,

$$\phi_i = a_i x_1 + b_i x_2 + c_i x_3. \quad (3.2)$$

Such an ansatz for massless scalar sources, guarantees the solution to be homogeneous and isotropic. According to AdS/CFT dictionary, massless scalar fields are dual to marginal operators of the corresponding field theory.

The relevant equations of motion for (3.1) can simply be obtained from (2.6) and there is an asymptotically AdS₅ black brane solution as (2.7) in which

$$f(\rho) = \frac{1 - \sqrt{1 - 4\lambda g(\rho)}}{2\lambda}, \quad (3.3)$$

where

$$g(\rho) = 1 - \frac{\alpha^2 \rho^2}{4} - m\rho^4, \quad m = \frac{1}{\rho_h^4} \left(1 - \frac{\alpha^2 \rho_h^2}{4} \right), \quad (3.4)$$

with ρ_h being the horizon radius and the constants a_i , b_i and c_i satisfy the following relations

$$\begin{aligned} \sum_{i=1}^3 a_i^2 &= \sum_{i=1}^3 b_i^2 = \sum_{i=1}^3 c_i^2 = \alpha^2, \\ \sum_{i=1}^3 a_i b_i &= \sum_{i=1}^3 a_i c_i = \sum_{i=1}^3 b_i c_i = 0. \end{aligned} \quad (3.5)$$

It is noted that $f(\rho_h) = 0$ and the UV boundary is defined as $\rho \rightarrow 0$ and the temperature of black brane is given by

$$T = \frac{1}{\pi \rho_h} \left(1 - \frac{\alpha^2 \rho_h^2}{8} \right). \quad (3.6)$$

There is an interesting feature for the momentum relaxation methods, *i.e.*, at the zero temperature one gets $f(\rho_h) = \frac{d}{d\rho} f(\rho)|_{\rho=\rho_h} = 0$, which is an extremal black brane. Although there is no $U(1)$ charge to produce an extremal black brane solution in this case, the momentum relaxation parameter gives us such feature similar to the case of RN-AdS black brane.

3.1 Holographic Entanglement Entropy

In the model that we are considering there are two deformations in the field theory due to the momentum relaxation parameter and GB terms. In this section, we find the corrections to the entanglement entropy because of these two deformations. To compute the entanglement entropy in this setup, we will use (2.13) and fix the coupling constants of higher-order terms according to five-dimensional GB gravity. Therefore, the entropy functional (2.13) reads as²

$$S = \frac{H^2 L_{AdS}^3}{4G_N} \int d\rho \frac{\sqrt{x'^2 + f^{-1}}}{\rho^3} \left(1 - 2\lambda \frac{f(fx'(2\rho x'' + 3x') + 3) - \rho f'}{(1 + fx'^2)^2} \right). \quad (3.7)$$

The next step is minimizing the entropy functional (3.7) in order to find the profile of the hypersurface which has been parametrized by $x(\rho)$. It is noted that $x(\rho)$ is supposed to be a smooth differentiable function with the condition $x(0) = \ell/2$. To proceed, one may consider the entropy functional as a one-dimensional action in which the corresponding Lagrangian is independent of $x(\rho)$ which leads to a conservation law. In other words, let us write (3.7) as $S = \int d\rho \mathcal{L}$, thus the equation of motion becomes

$$\frac{\partial}{\partial \rho} \left(\frac{\partial \mathcal{L}}{\partial x''} \right) - \left(\frac{\partial \mathcal{L}}{\partial x'} \right) = C, \quad \text{with} \quad \frac{\partial \mathcal{L}}{\partial x} = 0, \quad (3.8)$$

²The GB gravity is a special form of curvature squared action and it was shown that for five-dimensional GB gravity, the proposal of computing HEE presented in [25] reduces to [16, 24] and the results are the same. Note that taking into account the boundary term, only modifies the coefficient of leading UV-divergent term.

where C is a constant which can be fixed by imposing the condition that at the turning point ρ_t of the hypersurface in the bulk one has $x'(\rho_t) \rightarrow \infty$. After minimizing the functional of (3.7) and using the condition of the hypersurface turning point, one gets the following conserved quantity along the radial profile

$$x' \frac{1 + f(x'^2 - 2\lambda)}{f(f^{-1} + x'^2)^{3/2}} = \frac{\rho^3}{\rho_t^3}. \quad (3.9)$$

In principle, the above equation allows us to find $x'(\rho)$. In general, it is a difficult task to solve (3.9) to find a proper profile since it is a cubic equation for $x'(\rho)$. However, in some special cases, the semianalytic solutions might be obtained. In the following we will develop the behavior of HEE of a CFT whose states are in fact under the excitation of the both momentum relaxation and GB terms. Up to the leading order of λ , and after making use of the following expression

$$\frac{\ell}{2} = \int_0^{\rho_t} x'(\rho) d\rho, \quad (3.10)$$

one obtains

$$\ell = \frac{2\sqrt{\pi}(1 + \frac{3}{2}\lambda)\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})}\rho_t + \frac{1}{12}\alpha^2(1 - \frac{3}{2}\lambda)\rho_t^3, \quad (3.11)$$

which can be inverted to find the turning point of the proposed hypersurface in the bulk as follows

$$\rho_t = \frac{\Gamma(\frac{1}{6})}{2\sqrt{\pi}\Gamma(\frac{2}{3})}\ell - \frac{\Gamma(\frac{1}{6})^4}{192\pi^2\Gamma(\frac{2}{3})^4}\ell^3\alpha^2 + \left(\frac{-3\Gamma(\frac{1}{6})}{4\sqrt{\pi}\Gamma(\frac{2}{3})}\ell + \frac{5\Gamma(\frac{1}{6})^4}{128\pi^2\Gamma(\frac{2}{3})^4}\alpha^2\ell^3\right)\lambda + \mathcal{O}(\lambda^2, \alpha^3). \quad (3.12)$$

Plugging the results into (3.7), one gets the HEE as follows

$$S = \frac{H^2 L_{AdS}^3}{4G_N} \left(\frac{\mathfrak{a}}{\epsilon^2} + \mathfrak{b} \log \frac{\ell}{\epsilon} + \frac{\mathfrak{c}}{\ell^2} + \mathcal{O}(\lambda^2, \alpha^3) \right), \quad (3.13)$$

where ϵ stands for the UV scale which has been defined by the radial profile and

$$\begin{aligned} \mathfrak{a} &= 1 - \frac{13}{2}\lambda, \\ \mathfrak{b} &= \frac{1}{4}(1 - \frac{3}{2}\lambda)\alpha^2, \\ \mathfrak{c} &= -\frac{4\pi^{3/2}\Gamma(\frac{2}{3})^3}{\Gamma(\frac{1}{6})^3}(1 + \frac{9}{2}\lambda) + \frac{\ell^2}{4} \left((1 - \frac{3}{2}\lambda) \log \frac{\Gamma(\frac{1}{6})}{2^{2/3}\sqrt{\pi}\Gamma(\frac{2}{3})} - \frac{1}{3} + \frac{3}{4}\lambda \right) \alpha^2. \end{aligned} \quad (3.14)$$

The leading divergent term in (3.13) is in fact the usual area law; on the other hand, the logarithmic universal term in the HEE is found as follows

$$S_{\text{univ.}} = \frac{H^2 L_{AdS}^3}{16G_N} (1 - \frac{3}{2}\lambda) \alpha^2 \log \frac{\ell}{\epsilon}. \quad (3.15)$$

It is worth mentioning that for strip entangling region in $\text{CFT}_{d>2}$, in principle, there is no such a logarithmic universal term in the HEE. Nevertheless, in [14] it was observed that the universal term would appear due to momentum relaxation parameter. In our computation, the universal

term is appeared as well and according to (3.15), it has either of increasing or decreasing behavior depending on the sign of GB coupling.

To end up this section, we would like to mention that by setting $\alpha = 0$, the solution (2.7) turns to the five-dimensional GB-AdS black brane solution with a Ricci-flat horizon which was found in [36]. On the other hand, for low-excited state of CFT and near the UV boundary, (2.7) reduces to

$$ds^2 = \frac{\tilde{L}^2}{\rho^2} \left(-g(\rho)d\tau^2 + \frac{1}{g(\rho)}d\rho^2 + dX_1^2 + dX_2^2 + dX_3^2 \right), \quad (3.16)$$

where $g(\rho) = 1 - m\rho^4 + \mathcal{O}(m\lambda)$ and \tilde{L} can be considered as an effective AdS radius which is given by

$$\tilde{L}^2 = \frac{L_{AdS}^2}{f_\infty}, \quad \text{where } f_\infty = \frac{1 - \sqrt{1 - 4\lambda}}{2\lambda}. \quad (3.17)$$

Excited state due to such deformation in CFT is called thermal excitation. Thus, in particular, for the limit of $ml^4 \ll 1$, the change of entropy can be obtained via the following relation

$$\Delta S = \frac{H^2 \tilde{L}^2}{4G_N} \int d\rho \left(\frac{\delta \mathcal{S}}{\delta g} \right)_{|g=1} \Delta g, \quad (3.18)$$

where \mathcal{S} , up to the leading order of GB coupling, is the integrand of entropy function (3.7) in which $f(\rho)$ has been replaced by $g(\rho)$. Therefore, one obtains

$$\Delta S = S_{m \neq 0} - S_0 = \frac{H^2 \tilde{L}^3}{4G_N} \frac{(1 - 6\lambda f_\infty) \Gamma(\frac{1}{6})^2 \Gamma(\frac{1}{3})}{40\sqrt{\pi} \Gamma(\frac{2}{3})^2 \Gamma(\frac{5}{6})} m\ell^2, \quad (3.19)$$

where S_0 is the HEE for the vacuum case or pure AdS, namely $\alpha = \lambda = m = 0$, and it is given by

$$S_0 = \frac{H^2}{4G_N} \left(\frac{1}{\epsilon^2} - \frac{4}{\ell^2} \frac{\pi^{3/2} \Gamma(\frac{2}{3})^3}{\Gamma(\frac{1}{6})^3} \right). \quad (3.20)$$

The above HEE will reproduce the result in [18] for low-thermal excitation due to the GB term.

4 Holographic n -partite Information

In addition to entanglement entropy, the n -partite information is another useful quantity developed in the framework of quantum information theory that has conquered the drawbacks of the entanglement entropy, namely the existence of UV cutoff in the expression of entanglement entropy. In the case of two and three entangling regions, the n -partite information is equivalent to holographic mutual and tripartite information respectively which have their own characteristics as well; namely, they are regarded as criterion that indicates the amount of shared information, or more precisely the correlation, between the entangling regions [37]. Concerning the peculiarities of holographic mutual and tripartite information, it is worth investigating the effect of higher-order terms and momentum dissipation on these quantities which is the main task of this section.

4.1 Holographic Mutual Information

For two separated systems, *e.g.*, A_1 and A_2 , the mutual information would be a proper measure for quantifying the amount of entanglement (or information) that these two systems can share. It is shown that for two separated systems, the mutual information is a finite quantity and is given by [38]

$$I(A_1, A_2) = S(A_1) + S(A_2) - S(A_1 \cup A_2), \quad (4.1)$$

where $S(A_1 \cup A_2)$ is the entanglement entropy for the union of two entangling regions. Holographic mutual information undergoes a first order phase transition due to a discontinuity in its first derivative [39]. Holographically, this phase transition has in fact a simple explanation, *e.g.*, for the union of two strips with the same length ℓ separated by distance h , there are two different configurations which are the disconnected and connected ones, and the one with minimum area should be chosen; these configurations are schematically shown in Fig.1. Depending on the value of h/ℓ the corresponding minimal configurations, *i.e.*, RT surfaces, may change from one to another.



Figure 1: Schematic representation of two different configurations for computing the entanglement entropy of union of regions.

Transition of $S(A_1 \cup A_2)$ from $S_{\text{dis.}} = 2S(\ell)$ to $S_{\text{con.}} = S(2\ell + h) + S(h)$ and vice versa results in a phase transition in the mutual information. In other words, holographic mutual information vanishes or takes a finite value depending on the values of the entangling regions lengths and their separation. Therefore, the mutual information in AdS_5 background becomes

$$I(A_1, A_2) = \begin{cases} 2S(\ell) - S(h) - S(h + 2\ell), & 0 < \frac{h}{\ell} < r_1 \\ 0, & r_1 \leq \frac{h}{\ell} \end{cases} \quad (4.2)$$

and it can be shown that $r_1 = \sqrt{3} - 1$ or in other words, $h_{\text{crit.}} = (\sqrt{3} - 1)\ell$. The case is the same when only the higher-order GB term is included and $\alpha = 0$, but in presence of momentum relaxation parameter, this phase transition curve depends on α and λ parameters. In order to study the effect of mentioned parameters on this phase transition, let us first obtain the corrections to the mutual information. In the presence of momentum dissipation and GB terms, let us write the mutual information as

$$I(A_1, A_2) = I_0(A_1, A_2) + \Delta I(A_1, A_2), \quad (4.3)$$

where $I_0(A_1, A_2)$ stands for the mutual information when $\alpha = \lambda = 0$, and after making use of the corresponding entanglement entropies for ℓ , h and $2\ell + h$ regions from (3.13), it is given by

$$I_0 = \frac{H^2 L_{AdS}^3}{4G_N} i_0, \quad i_0 = \frac{4\pi^{3/2} \Gamma(\frac{2}{3})^3}{\Gamma(\frac{1}{6})^3} \left(\frac{1}{(2\ell + h)^2} + \frac{1}{h^2} - \frac{2}{\ell^2} \right). \quad (4.4)$$

On the other hand, the correction part becomes

$$\Delta I(A_1, A_2) = \frac{H^2 L_{AdS}^3}{4G_N} \left(\left(1 - \frac{3}{2}\lambda\right) \alpha^2 i_1 + \frac{9}{2} \lambda i_0 + \mathcal{O}(\lambda^2, \alpha^3) \right), \quad (4.5)$$

where one has

$$i_1 = \frac{1}{4} \log \frac{\ell^2}{h(2\ell + h)}. \quad (4.6)$$

We have plotted the normalized critical curve in Fig.2 as a function of momentum relaxation parameter α for three cases of $\lambda = 0$ and $\lambda = \pm 0.06$. It is noted that we have used $\ell = 1$ and by the normalized critical curve we mean $\tilde{r}_{crit.} = \frac{r_{crit.}}{r_{crit.}^{\alpha=0}}$.

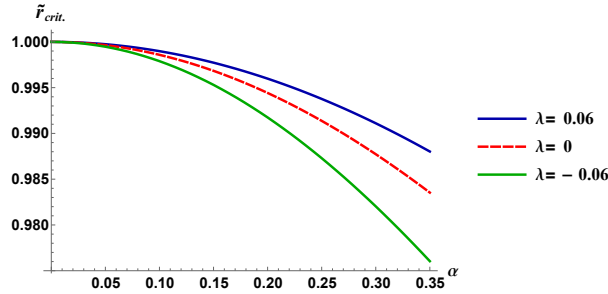


Figure 2: Normalized transition curve as a function of momentum relaxation parameter α for $\lambda = 0.06$ (upper curve), $\lambda = 0$ (middle curve) and $\lambda = -0.06$ (lower curve).

As one observes, the critical distance between two subsystems in which the phase transition happens, is indeed a decreasing function of both the momentum relaxation and GB coupling parameters. Specifically, by considering the effect of GB and momentum relaxation parameters on this critical distance, one concludes that although the general behavior of critical phase transition is decreasing, depending on the sign of GB coupling λ , it behaves differently; as it is shown in Fig.2, for $\lambda > 0$ the phase transition occurs in larger h comparing to the cases of $\lambda \leq 0$.

4.2 Holographic Tripartite Information

Besides mutual information, in a three-body system with topological order, tripartite information might be utilized as a quantity to characterize entanglement in states of the system. It was first

introduced in [40] as the topological entropy and defined by

$$I^{[3]}(A_1, A_2, A_3) = S(A_1) + S(A_2) + S(A_3) - S(A_1 \cup A_2) - S(A_1 \cup A_3) - S(A_2 \cup A_3) + S(A_1 \cup A_2 \cup A_3), \quad (4.7)$$

where $S(A_1, A_2, A_3)$ is the entanglement entropy for the union of three subsystems. It is shown that the tripartite information is always finite even when the regions share boundaries. To compute the holographic tripartite information by pursuing the RT proposal of finding the minimal surface, the union terms of $S(A_i \cup A_j)$ and $S(A_1, A_2, A_3)$ deserve to be discussed further. All possible diagrams for the non-mixed configurations of these terms are represented in Fig.3 noting that the rest of configurations can be obtained by rearranging these ones. It is worth mentioning that we have restricted ourselves to the case in which the entanglement entropy is an increasing function of entangling region. Therefore, the mixed configurations have not been considered [41].

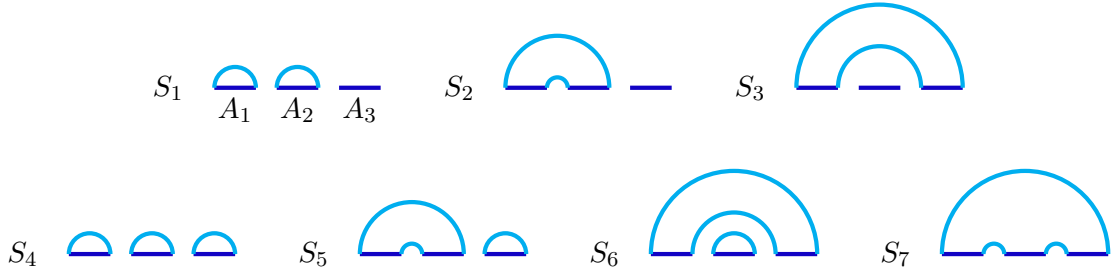


Figure 3: Schematic representation of competing configurations in the computation of $S(A_i \cup A_j)$ and $S(A_1, A_2, A_3)$.

Thus, $S(A_i \cup A_j)$ and $S(A_1, A_2, A_3)$ are given by the minimum among the following quantities

$$\begin{aligned} S(A_i \cup A_j) & \begin{cases} 2S(\ell) \equiv S_1 \\ S(2\ell + h) + S(h) \equiv S_2 \\ S(3\ell + 2h) + S(\ell + 2h) \equiv S_3 \end{cases} \\ S(A_1, A_2, A_3) & \begin{cases} 3S(\ell) \equiv S_4 \\ S(3\ell + 2h) + S(\ell + 2h) + S(\ell) \equiv S_5 \\ S(2\ell + h) + S(\ell) + S(h) \equiv S_6 \\ S(3\ell + 2h) + 2S(h) \equiv S_7 \end{cases} \end{aligned}$$

Therefore, one can write

$$I^{[3]}(A_1, A_2, A_3) = 3S(\ell) - 2 \min\{S_1, S_2\} - \min\{S_1, S_3\} + \min\{S_4, S_5, S_6, S_7\}. \quad (4.8)$$

As a special case when $\alpha = 0$, depending on the minimal area, the holographic tripartite information for three entangling regions with the same length ℓ separated by distance h is given by

$$I^{[3]}(A_1, A_2, A_3) = \begin{cases} S(\ell) - 2S(h + 2\ell) + S(2h + 3\ell), & 0 < \frac{h}{\ell} < r_1 \\ 2S(h) - 3S(\ell) + S(2h + 3\ell), & r_1 \leq \frac{h}{\ell} < r_2 \\ 0, & r_2 \leq \frac{h}{\ell} \end{cases} \quad (4.9)$$

which is the same as AdS_5 background and one finds $r_1 = \sqrt{3} - 1$ and $r_2 = \frac{\sqrt{7}-1}{2}$.

Similar to the mutual information, one can investigate that in presence of momentum relaxation parameter, the transition curves show a decreasing behavior with respect to α and λ and for positive (negative) value of λ , the phase transition in tripartite information happens in larger (smaller) ratio than the case of $\lambda = 0$.

We end this section with a comment on special property of tripartite information in holographic theories. Tripartite information can be written in terms of the mutual information as follows

$$I^{[3]}(A_1, A_2, A_3) = I(A_1 \cup A_2) + I(A_1 \cup A_3) - I(A_1, A_2 \cup A_3). \quad (4.10)$$

For arbitrary states of systems, tripartite information has no definite sign, namely depending on the underlying field theory, this quantity can be positive, negative or zero [38, 42, 43]. However, in strongly coupled CFTs with holographic duals it is argued that tripartite information is always negative [44, 45], and this property is related to the monogamy of the mutual information.³ In principle, it can be concluded that the holography leads to a constraint on this quantity and its sign might be employed in various works (see for example [46, 47]). In Fig.4, we have plotted the tripartite information as a function of momentum relaxation parameter and GB coupling and fixed values of entangling regions length. One observes that it always remains negative. This behavior also holds when one changes the length of entangling regions for the given (fixed) values of momentum relaxation and GB coupling parameters.

5 Conclusion

In this paper, we studied the effect of higher-order derivative terms on HEE in the theories with momentum relaxation parameter. Higher-order gravity theories are also interesting in a sense that they provide us with an effective description of quantum corrections and one may probe the finite coupling effects and the a - and c -theorems via making such corrections to the Einstein gravity theory in the bulk space. We used GB gravity theory as an example of higher-order derivative in five-dimensional space-time and for a strip entangling region, we obtained the corrections to universal and finite terms of HEE.

³In the context of quantum information theory, the inequality of the form $F(A_1, A_2) + F(A_1, A_3) \leq F(A_1, A_2 \cup A_3)$ is known as monogamy relation. This feature of measurement is related to the security of quantum cryptography indicating that entangled correlations between A_1 and A_2 cannot be shared with a third system A_3 without spoiling the original entanglement [42].

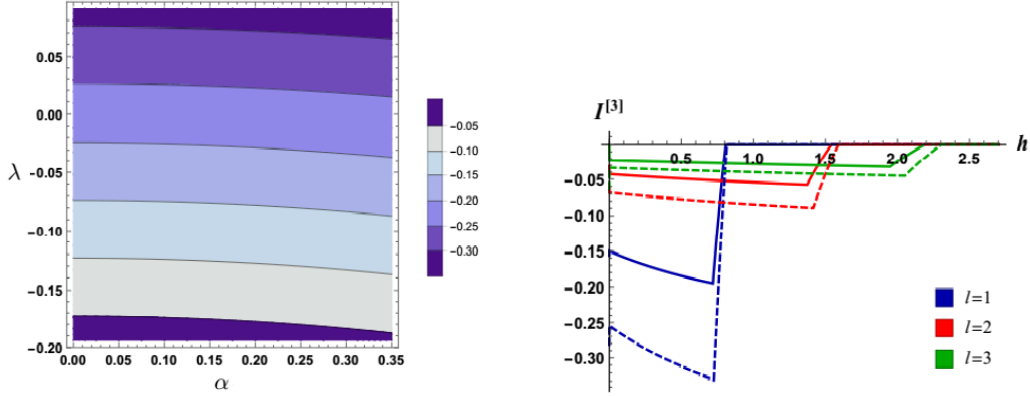


Figure 4: *Left plot:* Contour plot of tripartite information for $\ell = 1$ and $h = 0.3$. *Right plot:* Tripartite information for $\alpha = 0.3$, $\lambda = -0.06$ (solid curves) and $\lambda = 0.06$ (dashed curves).

In the context of quantum information theory and also quantum many-body systems, for two disjoint systems, the mutual information is usually used as a measure of quantum entanglement that these two systems can share; the mutual information can also be utilized as a useful probe to address certain phase transitions and critical behavior in these theories. For example, it is known that mutual information undergoes a transition beyond which it is identically zero; this kind of transition which is called as disentangling transition is in fact universal qualitative feature for all classes of theories with holographic duals [48]. In this paper we considered the effect of GB term on such phase transition in both of the mutual and tripartite information. Noting that the GB coupling is constrained to a small range, *i.e.* $-0.194 \lesssim \lambda \leq 0.09$ [49–51], it was shown that the behavior of such phase transition is different depending on the range of GB coupling. For $\lambda > 0$ this transition happens in larger value than the case of $\lambda < 0$. Furthermore, we showed that the tripartite information has negative value in our setup which means that mutual information is monogamous.

From the holographic point of view when the bulk theory is described by GB gravity for a strip geometry, in [21], a candidate for a c -function in arbitrary dimensions has been introduced; within this approach it might be more appropriate to consider this issue from (3.13). We leave further investigations to future works.

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